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THE MICROSCOPIC CALCULATION OF THREE-NUCLEON SYSTEM

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Abstract

In this research, the Faddeev technique is applied to compute the ground state binding energy of three-**nucleon** system. Yukawa type malflit-Tjon potential is used for nucleon-nucleon interaction. The three-nucleon binding energy for s wave ($\ell = 0$) interaction is 7.538 MeV and for $\ell = 0{,}2$ the binding energy becomes 7.550 MeV.

Keywords: Fadeev, three-boson, T-matrix.

Introduction

Historically, the study of the three-body problems is decisive importance for nuclear physics. The bound state of three nucleons is still not understood since the nuclear interactions cannot be calculated rigorously from an underlying theory. Therefore test of similar basic nature still lie in the future. At present one uses purely phenomenological forces based on meson theory [Machleidt R,1989] and adjusts them to describe two-nucleon observables. The question is then whether these forces are also sufficient to describe three interacting nucleons or whether in addition three-nucleon forces are needed [Friar J.L, Gibson B.F, and Payne G.L, 1984]. Since the three-body Schrödinger equation can be solved numerically in a precise manner, the three-nucleon system plays a very significant role in answering that question. In the three-body problem one has to face the geometrical difficulty of a three-body, spin- and iso-spin degrees of freedom and the violent variation of the nuclear force at short distances which induces high momentum components into the wave function.

Some other approaches used to treat three-body systems are variational calculations [Delves L.M, 1972], the use of hyperspherical harmonics [Fabre de la Ripelle M,1987], and the Green's function Monte Carlo technique [Schmidt K.E, 1987]. The Faddeev equations have been discussed extensively [Glockle W, 1983]. We shall present only the momentum space treatment of Faddeev equation. The momentum space is the natural one if one uses field theoretical potentials like the OBEP(one-boson-exchange potential)[Holinde K, Machleidt R,1975].

The three- and four-body problems present an interesting challenge to do research. Therefore, we are interested in the three-nucleon system (triton). Before going to solve real ^3_1H system, we will start with a simple system called three-boson system. In this case all three nucleons are considered as spinless particles. The real problem of three nucleons (triton) will be described in the following articles.

The Basic Definition for Three-Body System

In a three-body system there are three different two-body subsystems. We can choose one of them and if we choose the particle “1” as a spectator, only particle “2” and “3” will interact. There are relative motions of particle “1” to the center of mass position of the pair (2,3) and we call this state channel “1” and for example it is shown in Fig.(1).

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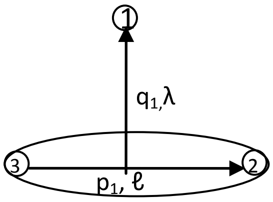
For channel “1”



$\mathbf{k_1}$ = momentum of particle 1
$\mathbf{k}_2$= momentum of particle 2
$\mathbf{k}_3$= momentum of particle 3

Figure 1 Particle configuration for channel “1”.

For identical particles, m1=m2=m3 and therefore

$$\mathbf{p}_1 = \frac{1}{2}(\mathbf{k}_2 - \mathbf{k}_3)\ ,\quad \mathbf{q}_1 = \frac{2}{3}\left\{\mathbf{k} - \frac{1}{2}(\mathbf{k}_2 + \mathbf{k}_3)\right\}$$

Generally, we can express for other channels as

$$\mathbf{p}_{\mathrm{i}} = \frac{1}{2}(\mathbf{k}_{\mathrm{j}} - \mathbf{k}_{\mathrm{k}})\ ,\quad \mathbf{q}_i = \frac{2}{3}\left\{\mathbf{k}_{\mathrm{i}} - \frac{1}{2}(\mathbf{k}_{\mathrm{j}} + \mathbf{k}_{\mathrm{k}})\right\}.$$

For the simplest case of three identical particles called bosons, we can write the state for channel “1” is $\left|\mathrm{pq}(\ell\lambda)\mathrm{LM}\right\rangle_1 \equiv \left|\mathrm{pq}\alpha\right\rangle_1$, where “p” is the relative momentum and “ℓ” is the relative orbital angular momentum of particle “2” and “3”, “q” is the relative momentum and “λ”is the orbital angular momentum of particle “1” with respect to the subsystem(2,3). The relative motion of three particles is conveniently described by Jacobi momenta.

$$\mathbf{p}_1 = \frac{\mathrm{m}_3\mathbf{k}_2 - \mathrm{m}_2\mathbf{k}_3}{\mathrm{m}_2 + \mathrm{m}_3}\ ,\quad \mathbf{q}_1 = \frac{(\mathrm{m}_2 + \mathrm{m}_3)\mathbf{k}_1 - \mathrm{m}_1(\mathbf{k}_2 + \mathbf{k}_3)}{\mathrm{m}_1 + \mathrm{m}_2 + \mathrm{m}_3}$$

For identical particles,

$$\mathbf{p}_1 = -\frac{1}{2}\mathbf{p}_2 + \frac{3}{4}\mathbf{q}_2\ ,\quad \mathbf{q}_1 = -\mathbf{p}_2 - \frac{1}{2}\mathbf{q}_2\ ,\quad \mathbf{p}_1 = -\frac{1}{2}\mathbf{p}_3 - \frac{3}{4}\mathbf{q}_3\ ,\quad \mathbf{q}_1 = \mathbf{p}_3 - \frac{1}{2}\mathbf{q}_3$$

and their states are $\left|\mathrm{p}_1\mathrm{q}_1\right\rangle_1$, $\left|\mathrm{p}_2\mathrm{q}_2\right\rangle_2$ and $\left|\mathrm{p}_3\mathrm{q}_3\right\rangle_3$.

The two-body subsystem can be transformed by the permutation operators P12P23 and P13P23 as

$$P_{12}P_{23}\left|\mathrm{pq}\right\rangle_1 = P_{12}P_{23}(1{,}23) = (2{,}31) = \left|\mathrm{pq}\right\rangle_2\ ,\quad P_{13}P_{23}\left|\mathrm{pq}\right\rangle_1 = P_{13}P_{23}(1{,}23) = (3{,}12) = \left|\mathrm{pq}\right\rangle_3$$

The operators have to be evaluated in the same type of basis vectors such as

$${}_1\left\langle \mathbf{p_1 q_1} \middle| \mathbf{p_2 q_2}\right\rangle_2 = {}_1\left\langle \mathbf{p_1 q_1} \middle| -\frac{1}{2}\mathbf{p_2} + \frac{3}{4}\mathbf{q_2}, -\mathbf{p_2} - \frac{1}{2}\mathbf{q_2}\right\rangle_1$$

$${}_1\left\langle \mathbf{p_1 q_1} \middle| \mathbf{p_3 q_3}\right\rangle_3 = {}_1\left\langle \mathbf{p_1 q_1} \middle| -\frac{1}{2}\mathbf{p_3} - \frac{3}{4}\mathbf{q_3}, \mathbf{p_3} - \frac{1}{2}\mathbf{q_3}\right\rangle_1$$

Three-Body Faddeev Equations

The Faddeev equations [Faddeev L.D, 1961] have been proven to be very useful and we shall concentrate on them in this paper. The Faddeev equations transcribe the content of the Schrödinger equation in a unique manner into a set of three coupled equations.

The Schrödinger equation for a three-body system is

$$(H_0 + \sum_{i=1}^{3} \mathrm{V}_i)\Psi = \mathrm{E}\Psi \tag{1}$$

where $V_i \equiv V_{jk}$, $\mathrm{i} \neq \mathrm{j} \neq \mathrm{k}$ (Interaction in two-body subsystem), H_0 is the kinetic energy of the relative motion for three particles.

The solution of Schrödinger equation is

$$\Psi = \frac{1}{\mathrm{E} - \mathrm{H}_0} \sum_{\mathrm{i}=1}^{3} \mathrm{V}_\mathrm{i}\Psi \tag{2}$$

$$\Psi = \mathrm{G}_0 \sum_{\mathrm{i}=1}^{3} \mathrm{V}_\mathrm{i}\Psi \tag{3}$$

where $G_0 \equiv \dfrac{1}{E - H_0}$ and G0 is free three-body propagator. Ψ is decomposed into 3-components and these are called Faddeev components.

$$\Psi = \sum_{\mathrm{i}=1}^{3} \psi_\mathrm{i} \tag{4}$$

We define

$$\psi_\mathrm{i} \equiv \mathrm{G}_0\mathrm{V}_\mathrm{i}\Psi \tag{5}$$

Eq.(4) is inserted for Ψ on the right hand side in the Eq.(5),

$$\psi_\mathrm{i} = \mathrm{G}_0\mathrm{V}_\mathrm{i} \sum_{\mathrm{j}=1}^{3} \psi_\mathrm{j} \tag{6}$$

simply rearrange Eq.(6) and becomes

$$(1 - \mathrm{G}_0\mathrm{V}_\mathrm{i})\psi_\mathrm{i} = \mathrm{G}_0\mathrm{V}_\mathrm{i} \sum_{\mathrm{j}\neq\mathrm{i}} \psi_\mathrm{j} \tag{7}$$

We can expand $(1 - \mathrm{G}_0\mathrm{V}_\mathrm{i})^{-1}$ by using binomial theorem

$$(1 - G_0V_i)^{-1}\, G_0V_i = (1 + \mathrm{G}_0\mathrm{V}_\mathrm{i} + \mathrm{G}_0\mathrm{V}_\mathrm{i}\mathrm{G}_0\mathrm{V}_\mathrm{i} + ---)\ \mathrm{G}_0\mathrm{V}_\mathrm{i}$$

$$= \mathrm{G}_0(\mathrm{V}_\mathrm{i} + \mathrm{V}_\mathrm{i}\mathrm{G}_0\mathrm{V}_\mathrm{i} + \mathrm{V}_\mathrm{i}\mathrm{G}_0\mathrm{V}_\mathrm{i}\mathrm{G}_0\mathrm{V}_\mathrm{i} + ---) = G_0T_i$$

We define T_i as follow

$$\mathrm{T}_\mathrm{i} = \mathrm{V}_\mathrm{i} + \mathrm{V}_\mathrm{i}\mathrm{G}_0\mathrm{T}_\mathrm{i} \tag{8}$$

where T_i is the two-body T operator for the pair i.

The Eq.(7) becomes

$$\psi_i = G_0 T_i \sum_{j \neq i} \psi_j \tag{9}$$

If we expand

$$\psi_1 = G_0 T_1 (\psi_2 + \psi_3) \tag{10}$$

$$\psi_2 = G_0 T_2 (\psi_3 + \psi_1) \tag{11}$$

$$\psi_3 = G_0 T_3 (\psi_1 + \psi_2) \tag{12}$$

This is a set of 3-coupled equation called Faddeev equation. For identical particles, the three equations can be reduced to a single one by using permutation operators. Therefore, the Faddeev equation for three identical particles is

$$\psi = G_0 TP\psi \tag{13}$$

where, P is the permutation operator and it is defined as

$$P \equiv P_{12}P_{23} + P_{13}P_{23} \tag{14}$$

The Faddeev Equation in Momentum Space for Three-Boson System

We consider the Faddeev equation (ψ=G0TP ψ) in the momentum space representation. We project ψ on the three particle basis state $\langle pq(\ell\lambda)LM| \equiv \langle pq\alpha|$

$$_1\langle pq\alpha|\psi\rangle_1 = {}_1\langle pq\alpha|G_0 TP\psi\rangle_1 \tag{15}$$

$$_1\langle pq\alpha|\psi\rangle_1 = \frac{1}{E - \frac{p^2}{m} - \frac{3q^2}{4m}} {}_1\langle pq\alpha|TP\psi\rangle_1 \tag{16}$$

Then, we insert the completeness relation

$$_1\langle pq\alpha|\psi\rangle_1 = \frac{1}{E - \frac{p^2}{m} - \frac{3q^2}{4m}} \sum_{\alpha'} \int p'^2 dp' \int q'^2 dq' \sum_{\alpha''} \int p''^2 dp'' \int q''^2 dq'' \, {}_1\langle pq\alpha|T|p'q'\alpha'\rangle_1$$

$$_1\langle p'q'\alpha'|P|p''q''\alpha''\rangle_{11}\langle p''q''\alpha''|\psi\rangle_1 \tag{17}$$

The two-body T-matrix term from the Eq.(17) must have the following condition

$$_1\langle pq\alpha|T|p'q'\alpha'\rangle_1 = \frac{\delta(q-q')}{qq'} \delta_{\alpha\alpha'} t_\ell (p, p', E - \frac{3q^2}{4m}) \tag{18}$$

For ground state, one has L=0 and consequently $\ell = \lambda$ and first we consider the simplest case of pure s-wave interaction (ℓ =0). The Faddeev component from the Eq.(17) is defined as

$$\langle pq(\ell=0,\lambda=0,L=0)\,M=0|\psi\rangle \equiv \psi(pq) \tag{19}$$

Evaluation of permutation operator P from the Eq.(17) is the purely geometrical problem.

$${}_1\langle p'q'\alpha'|P|p''q''\alpha''\rangle_1 = {}_1\langle p'q'\alpha'|(P_{12}P_{23}+P_{13}P_{23})|p''q''\alpha''\rangle_1 \tag{20}$$

$${}_1\langle p'q'\alpha'|P|p''q''\alpha''\rangle_1 = {}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_2 + {}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_3 \tag{21}$$

Again P13P23 = P23P12P23P23 and

$${}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_3 = {}_1\langle p'q'\alpha'|P_{23}P_{12}P_{23}P_{23}|p''q''\alpha''\rangle_1 \tag{22}$$

$${}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_3 = (-1)^{\ell}\,{}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_2(-1)^{\ell'} \tag{23}$$

For bosons both ℓ and ℓ' have to be even and therefore,

$${}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_3 = {}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_2 \tag{24}$$

Therefore Eq.(21) can be written as

$${}_1\langle p'q'\alpha'|P|p''q''\alpha''\rangle_1 = 2\,{}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_2 \tag{25}$$

For $(\ell = 0)$

$${}_1\langle p'q'\alpha'|p''q''\alpha''\rangle_2 = \frac{1}{2}\int_{-1}^{1} dx\,\frac{\delta(p'-\pi_1)}{p^2}\,\frac{\delta(p''-\pi_2)}{p''^2} \tag{26}$$

Then, Eq.(17) becomes

$$\psi(pq) = \frac{1}{E-\frac{p^2}{m}-\frac{3q^2}{4m}}\int q'^2dq'\int_{-1}^{1} dx\,t_0\left(p,\pi_1,E-\frac{3q^2}{4m}\right)\psi(\pi_2 q') \tag{27}$$

This integral equation in two variables is the three-boson Faddeev equation for pure s-wave interaction.

Again we consider the ℓ up to two. Generally for any ℓ

$$\begin{aligned}{}_1\langle pq\alpha|p'q'\alpha'\rangle_2 &= \delta_{LL'}\sum_{\ell_1+\ell_2=\ell}\ \sum_{\ell'_1+\ell'_2=\ell'}(-)^{\ell'}(\tfrac{1}{2})^{\ell'_1+\ell_2+1}\sqrt{\hat{\ell}\hat{\ell}'\hat{\lambda}\hat{\lambda}'}\,\frac{(q)^{\ell_2+\ell'_2}(q')^{\ell_1+\ell'_1}}{(\pi_1)^{\ell}(\pi_2)^{\ell'}}\\ &\sqrt{\frac{\hat{\ell}!\hat{\ell}'!}{(2\ell_1)!(2\ell_2)!(2\ell'_1)!(2\ell'_2)!}}\sum_f\sum_{f'}\begin{Bmatrix}\ell_1 & \ell_2 & \ell\\ \lambda & L & f\end{Bmatrix}\begin{Bmatrix}\ell'_2 & \ell'_1 & \ell'\\ \lambda & L & f'\end{Bmatrix}C(\ell_2\lambda f;00)C(\ell'_1\lambda' f';00)\\ &\sum_k P_k(x)\begin{Bmatrix}f & \ell_1 & L\\ f' & \ell_2 & k\end{Bmatrix}C(k\ell_1 f';00)C(k\ell'_2 f;00)\hat{k}\end{aligned} \tag{28}$$

where $\hat{\ell} = 2\ell+1$

$$\pi_1 = \sqrt{\tfrac{1}{4}q^2+q'^2+qq'x} \qquad \pi_2 = \sqrt{q^2+\tfrac{1}{4}q'^2+qq'x}$$

The Fadeev equation in momentum space representation for three-boson system becomes as the following equation

$$\psi_\alpha(pq)=\frac{1}{E-\frac{p^2}{m}-\frac{3q^2}{4m}}\sum_{\ell'}\int q'^2dq'\int_{-1}^{1}dx\, t_\ell(p,\pi_1,E-\frac{3q^2}{4m})\,G_{\alpha\alpha'}(q,q',x)\,\psi_{\alpha'}(\pi_2 q') \qquad (29)$$

where

$$G_{\alpha\alpha'}(q,q',x)=(-1)^{\ell'}\sum_{\ell_1+\ell_2=\ell}\;\sum_{\ell'_1+\ell'_2=\ell'}\frac{(q)^{\ell_2+\ell'_2}(q')^{\ell_1+\ell'_1}\left(\frac{1}{2}\right)^{\ell_2+\ell'_1}}{(\pi_1)^{\ell}(\pi_2)^{\ell'}}\sqrt{\frac{\hat{\ell}!\hat{\ell}'!}{(2\ell_1)!(2\ell_2)!(2\ell'_1)!(2\ell'_2)!}}$$

$$\sum_f\sum_{f'}\begin{Bmatrix}\ell_1 & \ell_2 & \ell\\ \lambda & L & f\end{Bmatrix}\begin{Bmatrix}\ell'_2 & \ell'_1 & \ell'\\ \lambda & L & f'\end{Bmatrix}C(\ell_2\lambda f;00)C(\ell'_1\lambda' f';00)$$

$$\sum_k P_k(x)\begin{Bmatrix}f & \ell_1 & L\\ f' & \ell_2 & k\end{Bmatrix}C(k\ell_1 f';00)C(k\ell'_2 f;00)\hat{k}$$

Calculation of Two-Body T-Matrix Embedded In three-body space

For studying the three-body system, we require to know the two-body off shell T-matrix. We have known T = V+VG0T and the two-body T-operator in the three particle basis state is

$$\langle pq\alpha|T|p'q'\alpha'\rangle=\langle pq\alpha|V|p'q'\alpha'\rangle+\langle pq\alpha|VG_0T|p'q'\alpha'\rangle \qquad (30)$$

We insert the completeness relation $\sum_\alpha\int p^2dp\int q^2dq|pq\alpha\rangle\langle pq\alpha|=1$ in the second term on the right hand side.

$$\langle pq\alpha|VG_0T|p'q'\alpha'\rangle=\sum_{\alpha''}\sum_{\alpha'''}\int p''^2dp''\int q''^2\,dq''\int p'''^2dp'''\int q'''^2\,dq'''\langle pq\alpha|V|p''q''\alpha''\rangle$$

$$\langle p''q''\alpha''|G_0|p'''q'''\alpha'''\rangle\,\langle p'''q'''\alpha'''|T|p'q'\alpha'\rangle$$

Then

$$\langle pq\alpha|VG_0T|p'q'\alpha'\rangle=\int p''^2dp''V_\ell(p,p'')\frac{1}{E-\frac{p''^2}{m}-\frac{3q^2}{4m}}\,\langle p''q\alpha|T|p'q'\alpha'\rangle \qquad (31)$$

The two-body T-operator in the three particles basis is clearly diagonal in the spectator quantum numbers "q" and "λ" but depends on "q" through the kinetic energy in G0. The energy available to the interacting two-body subsystem is $E-3q^2/4m$: consequently the T-operator is

$$\langle pq\alpha|T|p'q'\alpha'\rangle=\frac{\delta(q-q')}{qq'}\delta_{\alpha\alpha'}t_\ell\left(p,p',E-\frac{3q^2}{4m}\right) \qquad (32)$$

The two-body interaction in the three-particle basis is

$$\langle pq\alpha|V|p'q'\alpha'\rangle=\frac{\delta(q-q')}{qq'}\delta_{\alpha\alpha'}V_\ell(p,p') \qquad (33)$$

Then the two-body T-matrix for three-boson system is

$$t_\ell\left(p,p',E-\frac{3q^2}{4m}\right)=V_\ell(p,p')+\int p''^2dp''V_\ell(p,p'')\frac{1}{E-\frac{p''^2}{m}-\frac{3q^2}{4m}}t_\ell\left(p'',p',E-\frac{3q^2}{4m}\right) \quad (34)$$

This equation can be solved by using the Gauss Elimination Method.

Numerical Technique

The integral equation Eq.(35) is discretized in the variable "q". We have to choose the appropriate quadrature points "q" and "x". We introduce a cut-off value qmax and distribute properly Gauss-Legendre quadrature points over the intervals $0\le q\le q_{\max}$. But the skew arguments (π_2) in ψ under integral require an interpolation. The maximal value of π_2 is $3q_{\max}/2$. This fact is important in keeping the number of discretization points as low as possible. The two- body subsystem is controlled by the variable "p". The value $3q_{\max}/2$ is much lower than the typical cut-off value in p-variable beyond which $\psi(pq)$ can be neglected. Faddeev equation Eq.(35) is solved in that smaller interval.

We now consider an interpolation in the form

$$f(x)=\sum_k S_k(x)f(x_k) \quad (35)$$

where Sk(x) are known function and {xk} is a set of discrete grid point distributed over an interval in which function "f" has to be determined. We apply this form to the p-variable in the Faddeev equation Eq.(35).

$$\psi(p,q)=\frac{1}{E-\frac{p^2}{m}-\frac{3}{4m}q^2}\int_0^\infty q'^2dq'$$

$$\times\sum_m\sum_k\int_{-1}^{1}dx\,t_\ell\left(p,p_m,E-\tfrac{3}{4m}q^2\right)S_k(\pi_2)S_m(\pi_1)\psi(p_k,q') \quad (36)$$

Then we introduce Gaussian quadrature in the variable "q" and then

$$\psi(p_j,q_i)=\sum_\ell\sum_{ix} W_\ell\, q_\ell^2 W_{ix}\frac{1}{E-\frac{p^2}{m}-\frac{3}{4m}q^2}$$

$$\sum_m\sum_{k'} t_\ell\left(p,p_m,E-\tfrac{3}{4m}q^2\right)S_k(\pi_2)S_m(\pi_1)\psi(p_k,q') \quad (37)$$

This is the Faddeev equation for three-boson system and it can easily be solved numerically.

Numerical Accuracy

Our three-boson Faddeev equation is an integral equation and we will solve it numerically. And hence, we will present the convergence of the three-boson binding energy by varying the number of integration grid points. In order to solve the Faddeev components we have defined Nq is a number of discrete "q" points which represent the spectator momentum, Np is the number of grid points of relative momentum of two-body subsystem which is divided into two parts Np1 and Np2 and Nx is the corresponding number of x integration. We have taken two parts of integration limit in the momentum "p" range, the first interval is taken by 0.0fm^{-1} to $p_{\max} = \frac{3q_{\max}}{2} + 0.3\text{fm}^{-1}$ and the range of the second interval from $p_{\max}$ to p_{cut}. First we take Np1=10, Np2=10, Np=10, $q_{\max} = 5.0\text{fm}^{-1}$ and $p_{\text{cut}} = 40.00\text{fm}^{-1}$ arbitrarily and then we studied the precision of binding energy by varying the number of grid points Np1 in the first interval. We have found that the binding energy is converged to three decimal places at the number of grid point Np1=30. These results are displayed in Table 1.

Table 1 The convergence of binding energy by varying the number of grid point points Np1 in the first interval for $p_{\text{cut}} = 40\text{fm}^{-1}$.

Np_1	Np_2	Nq	Nx	q_{max}	p_{cut}	BE(MeV)
10	10	10	10	5.00	40.00	7.7732
20	10	10	10	5.00	40.00	7.5698
30	10	10	10	5.00	40.00	7.5675
40	10	10	10	5.00	40.00	7.5670

Then, we fixed the parameters of first interval and we studied the precision of binding energy by varying the number of grid point Np2 of the second interval. We observed that the binding energy is converged to four decimal places at the number of grid point Np2=16. The results are shown in Table 2.

Table 2 The convergence of binding energy by varying the number of grid point points Np_2 in the second interval for $p_{cut} = 40fm^{-1}$.

Np_1	Np_2	Nq	Nx	q_{max}	p_{cut}	BE(MeV)
30	**12**	10	10	5.00	40.00	7.5673
30	**14**	10	10	5.00	40.00	7.5673
30	**16**	**10**	**10**	**5.00**	**40.00**	**7.5672**
30	**18**	10	10	5.00	40.00	7.5672
30	**20**	10	10	5.00	40.00	7.5672

We fixed these data set such as Np1=30, Np2=16 and $p_{\text{cut}} = 40\text{fm}^{-1}$, then vary Nq, the binding energy is converged to four decimal places at Nq=20. The results are shown in Table 3.

Table 3 The convergence of binding energy by varying the number of grid point points Nq for $p_{cut} = 40fm^{-1}$.

Np_1	Np_2	Nq	Nx	q_{max}	p_{cut}	BE(MeV)
30	16	**16**	10	5.00	40.00	7.5369
30	16	**18**	10	5.00	40.00	7.5366
30	**16**	**20**	**10**	**5.00**	**40.00**	**7.5367**
30	16	**22**	10	5.00	40.00	7.5367
30	16	**24**	10	5.00	40.00	7.5367

Then we increase the p_{cut} value to $50.0fm^{-1}$. By varying the number of grid points Np2, the binding energy is converged to 7.5376 MeV at Np2=16 which is shown in Table 4.

Table 4 The convergence of binding energy by varying the number of grid point points Np_2 in the second interval with $p_{cut} = 50fm^{-1}$.

Np_1	Np_2	Nq	Nx	q_{max}	p_{cut}	BE(MeV)
30	**16**	**10**	**10**	**5.00**	**50.00**	**7.5376**
30	**18**	10	10	5.00	50.00	7.5376
30	**20**	10	10	5.00	50.00	7.5376
30	**22**	10	10	5.00	50.00	7.5376
30	**24**	10	10	5.00	50.00	7.5376

Again we increase the q_{max} value to $7.00fm^{-1}$. We studied the precision of binding energy by varying the number of grid point Np1 and Np2 and Nq as the same procedure given above (Here we will not show the tables to save the page). The binding energy is converged to 7.5376 MeV and we have found that a small change in binding energy in the comparing the result of $q_{max} = 5.00fm^{-1}$. So we decide $q_{max} = 5.00fm^{-1}$ is enough for that system. Therefore we choose the parameter set of Np1=30, Np2=16, Nq=20, $p_{cut} = 50.00fm^{-1}$ and $q_{max} = 5.00fm^{-1}$ for that system and the binding energy of three-boson system is 7.5376MeV.

Result and discussion

We have found that the binding energy of the ground state of three-boson system is 7.538 MeV for pure s-wave interaction (ℓ=0) and when we increase ℓ up to two, the binding energy becomes 7.550 MeV. Our result does not agree with the experimental value of triton binding energy 8.48 MeV. The disagreement of our result and experimental value can be the following reasons: the two-body potential which we have used is not realistic potential and our system is not the realistic description of three-nucleon problem, it has been reduced to be a simple problem say three-boson system. We expect that our result will nearly agree with the experimental value if we include the spin and iso-spin. The realistic three-nucleon (triton) system will be presented in the upcoming paper.

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